On the second spherical harmonics of the cosmic ray angular distribution

1975 J. Phys. A: Math. Gen. 8 1349

(http://iopscience.iop.org/0305-4470/8/8/019)

J. Phys. A: Math. Gen., Vol. 8, No. 8, 1975. Printed in Great Britain.

On the second spherical harmonics of the cosmic ray angular distribution

J Kóta

Department of Cosmic Rays, Central Research Institute for Physics, Budapest, Hungary

Received 18 February 1975

Abstract. Explicit expressions describing the semi-diurnal variation of cosmic ray intensity are obtained in the frame of the convection–diffusion theory. Convection–diffusion equations are extended by introducing a symmetric traceless tensor accounting for the second harmonics of the cosmic ray angular distribution. Starting from the statistical Boltzmann equation a new transport equation is deduced which relates the second harmonics of the angular distribution to the gradients of cosmic ray streaming. Thus, compared to models used hitherto, a more quantitative calculation is carried out. Expressing the free space anisotropy in terms of geographical coordinates, the predicted diurnal and semi-diurnal variations are given. The results obtained are essentially in agreement with those of Quenby and Lietti but also differences arise as a result of the bending and divergence of the large-scale interplanetary magnetic field lines. Arguments are brought forward that a sunward cosmic ray streaming along the interplanetary field lines gives rise to a pitch angle distribution with an excess of particles of large pitch angles.

1. Introduction

Having entered the solar system, the Galactic cosmic radiation is subject to solar modulation which reduces its intensity and changes its angular distribution by introducing a solar-bound anisotropy as well. Due to the rotation of the earth the anisotropic directional distribution manifests itself in diurnal, semi-diurnal, etc cosmic ray intensity variations. Among these, the first daily harmonic has a satisfactory explanation in the convection–diffusion theory developed by Parker (1964) and Gleeson and Axford (1967). The semi-diurnal variation, however, is caused by the second and higher harmonics of the free space cosmic ray distribution; thus, in contrast to the first daily harmonic, it cannot be treated by convection–diffusion equations involving particle density and current density only (the latter corresponding to the first harmonic of the cosmic ray distribution).

Using different approaches, several authors (Quenby and Lietti 1968, Subramanian and Sarabhai 1967) have pointed out that the semi-diurnal variation arises as a result of the change of the cosmic ray density gradient, ie as a result of the second space derivative of cosmic ray density. Spiralling around the interplanetary magnetic field lines, particles perform several turns until being scattered. Thus the flux of particles arriving at the earth from a specific direction reflects cosmic ray density at the guiding centre of particle trajectory belonging to the given direction. Provided that the cosmic ray density is higher both above and below the ecliptic plane than in the plane itself, ie a nonzero second derivative of cosmic ray density exists (which is sometimes referred to as the bi-directional gradient), a semi-diurnal variation of cosmic ray intensity results with intensity maxima at about 3 and 15 h local time corresponding to the directions lying in the ecliptic plane perpendicularly to the interplanetary magnetic field lines. With increasing rigidity particles get farther from the ecliptic plane, giving rise to an increasing semi-diurnal wave.

In the present work the semi-diurnal variation is derived in terms of the convection–diffusion model. In order to achieve this, convection–diffusion theory will be extended to include the second harmonics of the cosmic ray angular distribution too. Starting from the statistical Boltzmann equation, an additional transport equation will be obtained which brings the second harmonics of the distribution into relation with the cosmic ray stream, suggesting that second harmonics are generated by the gradients of the stream. Different components of cosmic ray streams will be considered to generate second harmonics in this indirect way. Finally, expressing the free space anisotropy in terms of geographical coordinates, explicit expressions of the resulting diurnal and semi-diurnal variations will be given.

2. General equations

In its usual form the convection–diffusion theory considers the cosmic ray density and net particle flux. The latter, being a vector, is responsible for the anisotropy and results in a sinusoidal daily wave. However, it cannot give rise to semi-diurnal variation which is produced by the second (and higher) moments of the cosmic ray distribution. In this section we introduce a symmetric traceless tensor accounting for the second harmonics of the cosmic ray distribution and establish a new transport equation. It will be found that, as expected, the convection–diffusion equations remain virtually unaltered, their change due to the newly introduced tensor being negligible at least in cases where the angular distribution is not far from isotropy. On the other hand the new transport equation establishes a connection between the second harmonics and the cosmic ray flow. It will turn out that, like the cosmic ray flow, the quadrupole moment of the distribution can also be divided into convective and diffusive terms.

2.1. Moments of the cosmic ray distribution

When investigating cosmic ray distribution, one starts from the statistical Boltzmann equation

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial x_i}(\dot{x}_i f)+\frac{\partial}{\partial p_i}(\dot{p}_i f) = \left(\frac{\delta f}{\delta t}\right)_{\text{coll}} \tag{1}$$

where $f(x_i, p_i, t)$ is the distribution function and x_i and p_i ($i = 1, 2, 3$) represent the coordinates and components of momentum respectively. The electromagnetic Lorentz force is responsible for $\dot{p}_i$ while the right-hand side term accounts for the change of f due to scattering in the irregular magnetic field. Double indices indicate sums throughout the paper.

In order to obtain moments of the Boltzmann equation, the method developed by Gleeson and Axford (1967) is used but also the second harmonics of the distribution are included, ie f is assumed to be of the form

$$f(x_i, p_i, t) = f^{(0)}(x_i, p, t)+\boldsymbol{f}^{(1)}(x_i, p, t)\boldsymbol{e}+\boldsymbol{e}\boldsymbol{f}^{(2)}(x_i, p, t)\boldsymbol{e} \tag{2}$$

where $f^{(0)}$, $\boldsymbol{f}^{(1)}$ and $\boldsymbol{f}^{(2)}$ are scalar, vector and tensor respectively. $f^{(0)}$, $\boldsymbol{f}^{(1)}$ and $\boldsymbol{f}^{(2)}$ are independent of the direction of the momentum, the dependence on which appears in the unit vector, $\boldsymbol{e}$, pointing in the momentum's direction. In order to avoid ambiguities $\boldsymbol{f}^{(2)}$ is defined to be symmetric and traceless. Obviously an antisymmetric term would give no contribution to f. On the other hand the contribution of a unit tensor is direction-independent so it can be absorbed into $f^{(0)}$. The expansion used here is identical to that in spherical harmonics which is successfully applied for nearly isotropic distribution. The components of $\boldsymbol{f}^{(1)}$ and $\boldsymbol{f}^{(2)}$ are uniquely related to spherical harmonics of the first and second order respectively. The tensor $\boldsymbol{f}^{(2)}$ has five independent components corresponding to the five spherical harmonics of second order.

The usual particle number density U and net flux $\boldsymbol{S}$ can easily be obtained by using the form of the distribution function (equation (2)) and integrating over the direction of the momentum:

$$U(x_i, p, t) = \int fp^2 \,\mathrm{d}\Omega = 4\pi p^2 f^{(0)} \tag{3a}$$

$$S_j(x_i, p, t) = \int v_j fp^2 \,\mathrm{d}\Omega = \tfrac{4}{3}\pi p^2 v f_j^{(1)} \tag{3b}$$

where $\int \mathrm{d}\Omega$ represents integration over the direction of the momentum and $\boldsymbol{v}$ is the particle velocity.

In an analogous way let us define a symmetric traceless Q_{jk} as

$$Q_{jk}(x_i, p, t) = \int (v_j v_k - \tfrac{1}{3}v^2\,\delta_{jk}) fp^2 \,\mathrm{d}\Omega = \tfrac{8}{15}\pi p^2 v^2 f_{jk}^{(2)}. \tag{3c}$$

Q_{jk} may be referred to as the quadrupole moment of the cosmic ray distribution. Obviously the semi-diurnal variation is connected with Q_{jk}.

2.2. *Scattering mean free paths*

The collision term on the right-hand side of the Boltzmann equation can be evaluated using the method developed by Gleeson and Axford (1967). First, a Lorentz transformation leading into the frame moving with the solar wind is carried out; then, after having considered the effect of an inelastic scattering of cosmic ray particles on magnetic field inhomogeneities carried by the solar wind, results are transformed back into the fixed frame. In the convection–diffusion model a relaxation time approximation is used, ie scattering is characterized by a mean free path λ travelled by particles until scattering becomes isotropic. Here we adopt a model of subsequent independent scatterings. Having travelled a mean distance, l, particles are deflected with an angle ψ. The scattering need not be isotropic but it is described with a deflection angle distribution $\sigma(\psi)$, ie the scattering process is absorbed into $\sigma(\psi)$. Then it is found that rates at which the first and second harmonics of the anisotropy decay may be different. The calculation yields the mean free paths:

$$\frac{1}{\lambda_1} = \frac{1}{l}\int_0^{\pi} \tfrac{1}{2}(1-\cos\psi)\sigma(\psi)\sin\psi\,\mathrm{d}\psi \tag{4a}$$

$$\frac{1}{\lambda_2} = \frac{1}{l}\int_0^{\pi} \tfrac{3}{4}(1-\cos^2\psi)\sigma(\psi)\sin\psi\,\mathrm{d}\psi \tag{4b}$$

where λ_1 and λ_2 are the mean free paths belonging to the first and second harmonics of the anisotropy respectively.

In the real physical case diffusion cannot be treated as the result of subsequent separate scatterings, but the power spectrum of the irregular magnetic field is of importance (Jokipii 1966, Quenby 1973). Yet formulae (4*a*, *b*) remain applicable at high rigidities where the deflection in a coherent region of the irregular field is small. In this case l and ψ become the mean size of a coherent region and the deflection of particles from their ideal unperturbed trajectories in that region respectively.

It can easily be seen that $\lambda_1 = \lambda_2 = l$ for isotropic scattering (ie $\sigma(\psi)$ = constant). At high rigidities, however, $\sigma(\psi)$ is expected to be strongly peaked at $\psi = 0$. In this case it follows that

$$\frac{1}{\lambda_2} = \frac{1}{l}\int_0^\pi 3\frac{(1+\cos\psi)}{2}\frac{(1-\cos\psi)}{2}\sigma(\psi)\sin\psi\,\mathrm{d}\psi \simeq \frac{3}{\lambda_1} \tag{5}$$

since $1+\cos\psi \simeq 2$ can be taken in the integral. This result implies that higher harmonics describing smaller details of the distribution decay faster.

2.3. *Transport equations*

At this stage our aim is to obtain equations connecting the quantities U, $\boldsymbol{S}$ and $\boldsymbol{Q}$. To achieve this the moments of the Boltzmann equation (1) are to be considered. In investigating steady-state conditions, time derivatives can be ignored; furthermore, because of the high conductivity of the solar wind plasma,

$$\boldsymbol{E} = -\frac{1}{c}(\boldsymbol{V}\times\boldsymbol{B}) \tag{6}$$

can be substituted where $\boldsymbol{V}$ is the solar wind velocity and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field strengths respectively. Then assuming nearly isotropic distribution (ie $f^{(0)} \gg |\boldsymbol{f}^{(1)}|, |\boldsymbol{f}^{(2)}|$) and neglecting higher-order terms of V/v, calculation yields the equations

$$\frac{\partial S_r}{\partial x_r} = \frac{V_r}{3}\frac{\partial}{\partial x_r}\frac{\partial}{\partial p}(Up) \tag{7a}$$

$$\frac{3}{v\lambda_1}\left(\delta_{ir} + \frac{Ze\lambda_1}{pc}\epsilon_{isr}B_s\right)\left[S_r + \frac{p^3}{3}\frac{\partial}{\partial p}\left(\frac{U}{p^2}\right)V_r\right] = -\frac{\partial U}{\partial x_i} \tag{7b}$$

$$\begin{aligned}\frac{5}{v\lambda_2}&\left(\delta_{iq}\delta_{kr} + \frac{Ze\lambda_2}{pc}(\delta_{iq}\epsilon_{ksr}B_s + \delta_{kr}\epsilon_{isq}B_s)\right)\\ &\times\left\{Q_{rq} + \frac{p^4 v}{5}\frac{\partial}{\partial p}\left[\frac{1}{3v}\frac{\partial}{\partial p}\left(\frac{U}{p^2}\right)\left(V_rV_q - \frac{V^2}{3}\delta_{rq}\right)\right] + V_rS_q + V_qS_r - \frac{2}{3}V_tS_t\delta_{rq}\right\}\\ &= -\left(\frac{\partial S_i}{\partial x_k} + \frac{\partial S_k}{\partial x_i} - \frac{2}{3}\frac{\partial S_t}{\partial x_t}\delta_{ik}\right) + \frac{p^3}{3}\frac{\partial}{\partial p}\left[\frac{1}{p^2}\left(V_i\frac{\partial U}{\partial x_k} + V_k\frac{\partial U}{\partial x_i} - \frac{2}{3}V_t\frac{\partial U}{\partial x_t}\delta_{ik}\right)\right]\end{aligned} \tag{7c}$$

where Ze is the electric charge of the particle and ϵ_{ijk} represents the antisymmetric unit tensor.

It can be seen that equations (7*a*, *b*) are equivalent to the usual convection–diffusion equations.

$$\frac{3}{v\lambda_1}\left(\delta_{ir}+\frac{Ze\lambda_1}{pc}\epsilon_{isr}B_s\right)$$

is the inverse diffusion tensor and

$$-\frac{p^3}{3}\frac{\partial}{\partial p}\left(\frac{U}{p^2}\right)V$$

gives the convective streaming. $\boldsymbol{S}$ can be explicitly expressed from equation (7*b*) and is found to consist of convective and diffusive terms. The situation is closely analogous in the case of equation (7*c*), as will be presently shown. For in a shorthand notation equation (7*c*) can be written as

$$D_{ikqr}(Q_{rq}-Q_{rq}^{(\mathrm{conv})}) = -J_{ik} \tag{8}$$

where

$$D_{ikqr} = \frac{5}{v\lambda_2}\left(\delta_{iq}\,\delta_{kr}+\frac{Ze\lambda_2}{pc}(\delta_{iq}\epsilon_{ksr}B_s+\delta_{kr}\epsilon_{isq}B_s)\right) \tag{8a}$$

$$Q_{rq}^{(\mathrm{conv})} = -\frac{p^4 v}{5}\frac{\partial}{\partial p}\left[\frac{1}{3v}\frac{\partial}{\partial p}\left(\frac{U}{p^2}\right)\left(V_rV_q-\frac{V^2}{3}\delta_{rq}\right)\right]+V_rS_q+S_rV_q-\frac{2}{3}V_tS_t\,\delta_{rq} \tag{8b}$$

$$J_{ik} = \frac{\partial S_i}{\partial x_k}+\frac{\partial S_k}{\partial x_i}-\frac{2}{3}\frac{\partial S_t}{\partial x_t}\delta_{ik}+\frac{p^3}{3}\frac{\partial}{\partial p}\left[\frac{1}{p^2}\left(V_i\frac{\partial U}{\partial x_k}+V_k\frac{\partial U}{\partial x_i}-\frac{2}{3}V_t\frac{\partial U}{\partial x_t}\delta_{ik}\right)\right]. \tag{8c}$$

On the basis of (8),

$$Q_{ik} = Q_{ik}^{(\mathrm{conv})}-D_{ikqr}^{-1}J_{rq}; \tag{9}$$

$Q_{ik}^{(\mathrm{conv})}$ is connected with solar wind velocity and arises as a result of the motion of the scattering media; thus it may be regarded as being the convective part of Q_{ik}. On the other hand the second term on the right-hand side of (9) may be called diffusive since it is produced by gradients of the cosmic ray particle density and current density. The tensor D_{ikqr}^{-1} containing the mean free path λ_2 corresponds to the diffusion tensor.

The reference system appropriate for inverting D_{ikqr} is chosen so that the x axis points in the direction of the magnetic field, and the y and z axes are perpendicular to the x axis and to each other too. Then calculation yields

$$Q_{xx} = Q_{xx}^{(\mathrm{conv})}-\frac{v\lambda_2}{5}J_{xx} \tag{10a}$$

$$Q_{xy} = Q_{yx} = Q_{xy}^{(\mathrm{conv})}-\frac{v\lambda_2}{5}\frac{J_{xy}+KJ_{xz}}{1+K^2} \tag{10b}$$

$$Q_{xz} = Q_{zx} = Q_{xz}^{(\mathrm{conv})}-\frac{v\lambda_2}{5}\frac{J_{xz}-KJ_{xy}}{1+K^2} \tag{10c}$$

$$Q_{yy} = Q_{yy}^{(\mathrm{conv})}-\frac{v\lambda_2}{5}\frac{J_{yy}+2KJ_{yz}+2K^2(J_{yy}+J_{zz})}{1+4K^2} \tag{10d}$$

$$Q_{yz} = Q_{zy} = Q_{yz}^{(\mathrm{conv})}-\frac{v\lambda_2}{5}\frac{J_{yz}-K(J_{yy}-J_{zz})}{1+4K^2} \tag{10e}$$

$$Q_{zz} = Q_{zz}^{(\mathrm{conv})} - \frac{v\lambda_2}{5} \frac{J_{zz} - 2KJ_{yz} + 2K^2(J_{yy} + J_{zz})}{1 + 4K^2} \tag{10f}$$

where $K = Ze\lambda_2/pc = \lambda_2/R$, R being the gyroradius in the large-scale magnetic field. These results show that, as in the case of diffusive streaming, the diffusion is unaffected by the magnetic field in the direction of the field and is restricted in perpendicular directions.

Since the solar wind velocity is small with respect to particle velocities ($V/v \simeq 10^{-3}$), terms resulting in effects of the order of $(V/v)^2$ cannot be observed experimentally. Ignoring such terms equation (7c) reduces to

$$Q_{ik} = -D_{ikqr}^{-1} J_{rq}^* \tag{11a}$$

where

$$J_{rq}^* = \frac{\partial S_r}{\partial x_q} + \frac{\partial S_q}{\partial x_r} - \frac{2}{3} \frac{\partial S_t}{\partial x_t} \delta_{qr}. \tag{11b}$$

3. Second harmonics of the free space anisotropy

Inspection of equations (11a, b) shows that second harmonics of the anisotropy are produced by spacial gradients of the cosmic ray flow. In this section the co-rotation, the most dominant of the cosmic ray streamings, will be investigated and found to give rise to negligible semi-diurnal variation. The co-rotation apart, additional cosmic ray flows arise as a result of the solar zenith angle density gradient. These flows will turn out to produce the second harmonics of the cosmic ray anisotropy.

3.1. The effect of co-rotation

It has been shown by several authors (Parker 1964, for second-order effects see Somogyi 1972) that the rotating interplanetary magnetic field gives rise to a rigid co-rotation of cosmic radiation. Thus the co-rotational stream is

$$\boldsymbol{S} = \frac{\mu+2}{3} \boldsymbol{V}^{(\mathrm{cor})} U = \frac{\mu+2}{3} (\boldsymbol{\Omega} \times \boldsymbol{r}) U \tag{12}$$

where μ is the negative exponent of the cosmic ray energy spectrum, $\boldsymbol{\Omega}$ is the angular velocity vector of the sun and $\boldsymbol{r}$ is the radial vector pointing to the earth from the sun ($V^{\mathrm{cor}} \simeq 400\ \mathrm{km\ s^{-1}}$). Substituting equation (12) into equation (11b), we arrive at

$$J_{ik}^* = \frac{\mu+2}{3} \left(V_i^{\mathrm{cor}} \frac{\partial U}{\partial x_k} + V_k^{\mathrm{cor}} \frac{\partial U}{\partial x_i} \right) \tag{13}$$

which gives rise to a semi-diurnal amplitude of the order of $(V/v)^2$. Thus co-rotation produces no observable semi-diurnal variation.

3.2. The effect of zenith angle density gradient

At higher heliolatitudes particles travel shorter distances along the bent interplanetary magnetic field lines so that a solar zenith angle density gradient is produced (Quenby and Lietti 1968). Here, and in what follows, it is assumed that the regular spiralling motion of particles is dominant with respect to the diffusion, ie $R \ll \lambda$, R being the gyroradius. The zenith angle density gradient will produce two kinds of cosmic ray streaming (Parker 1965).

(i) Particle streaming perpendicular both to the density gradient and the interplanetary magnetic field. This streaming is connected with the regular spiralling motion and its magnitude is

$$\frac{Rv}{3r}\frac{\partial U}{\partial \theta},$$

where θ is the zenith angle and r is the distance from the sun.

(ii) In the direction opposite to the density gradient a diffusive stream will flow with a magnitude of

$$\frac{R^2 v}{3\lambda_1 r}\frac{\partial U}{\partial \theta}.$$

(iii) Quenby and Hashim (1969) have pointed out that cosmic ray flow of type (ii) is directed toward the ecliptic plane both below and above the plane (provided that the ecliptic plane is the plane of symmetry); thus conservation of the number of particles demands an outward flow along the magnetic field lines, the magnitude of which is given by

$$S_{\parallel} = \frac{v}{3r^2 \cos\psi}\int_0^r \frac{R^2}{\lambda_1}\frac{\partial^2 U}{\partial \theta^2}\,\mathrm{d}r' = \eta\frac{v}{3r\cos\psi}\frac{R^2}{\lambda_1}\frac{\partial^2 U}{\partial \theta^2} \tag{14}$$

where ψ is the garden hose angle ($\psi = 45°$ at the orbit of the earth) and η is a numerical factor that is determined by the dependence of the mean free path λ_1 on the distance from the sun. Assuming $\lambda \propto r^{\beta}$,

$$\eta = \int_0^1 \frac{x^{4-\beta}}{1+x^2}\,\mathrm{d}x$$

(we note that $\eta = 0{\cdot}12$ for $\beta = 0$ and $\eta = 0{\cdot}22$ for $\beta = 2$).

The first two of the three cosmic ray streams mentioned above actually vanish in the plane of symmetry, yet they have an important role in generating second harmonics.

The reference system appropriate to the calculations is chosen so that the x axis points in the direction of the interplanetary field, the z axis points in the zenith angle direction and the y axis is perpendicular to the x and z axes (see figure 1).

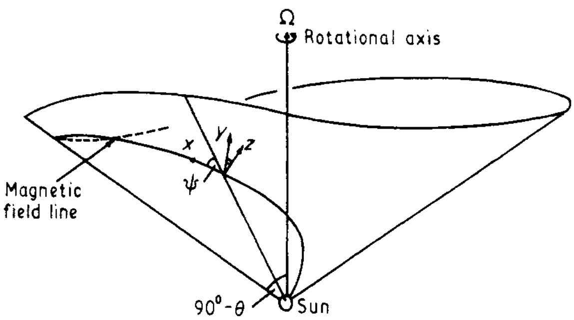

Figure 1. The reference system used in calculations. Magnetic field lines are wound on a cone around the rotational axis of the sun. Axis x is chosen to point along the field lines, axis y lies in the surface of the cone perpendicularly to field lines, while axis z points in the zenith angle direction normal to the surface of the cone.

Obviously the second harmonics of the cosmic ray anisotropy are given by the ratio of elements of $f^{(2)}$ to $f^{(0)}$ (see equation (2)). Using equations (10*a*–*f*), (11*a*, *b*) and considering the three cosmic ray streamings mentioned above, calculation yields

$$A = \frac{f^{(2)}_{xx}}{f^{(0)}} = \frac{15Q_{xx}}{2v^2U} = -2\frac{\lambda_2}{\lambda_1}\frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial\theta^2}+\lambda_2\frac{2-\sin^2\psi}{r}\frac{3S_\parallel}{vU} \tag{15a}$$

$$B = \frac{f^{(2)}_{xy}}{f^{(0)}} = 0 \tag{15b}$$

$$C = \frac{f^{(2)}_{yy}}{f^{(0)}} = \left(\frac{1}{2}+\frac{\lambda_2}{\lambda_1}\right)\frac{R^2}{2r^2U}\frac{\partial^2 U}{\partial\theta^2}-\lambda_2\frac{2-\sin^2\psi}{2r}\frac{3S_\parallel}{vU} \tag{15c}$$

$$D = \frac{f^{(2)}_{zz}}{f^{(0)}} = \left(-\frac{1}{2}+\frac{\lambda_2}{\lambda_1}\right)\frac{R^2}{2r^2U}\frac{\partial^2 U}{\partial\theta^2}-\lambda_2\frac{2-\sin^2\psi}{2r}\frac{3S_\parallel}{vU} \tag{15d}$$

$$E = \frac{f^{(2)}_{xz}}{f^{(0)}} = 0 \tag{15e}$$

$$F = \frac{f^{(2)}_{yz}}{f^{(0)}} = 0 \tag{15f}$$

where $S_\parallel$ is the cosmic ray stream in the direction of the magnetic field (see equation (14)). Terms containing $S_\parallel$ arise as a result of the divergence of interplanetary magnetic field lines while terms in C and D containing no λ_2/λ_1 are produced by a stream of type (i). Obviously positive A, C and D values result in intensity maxima from the x, y and z directions respectively.

At high rigidities $\lambda_2 = \frac{1}{3}\lambda_1$ can be taken (see equation (5)). Furthermore, assuming $\lambda \propto r^2$, ie $\beta = 2$, the results (15*a*–*f*) reduce to

$$A = -0{\cdot}44\frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial\theta^2} \tag{16a}$$

$$C = 0{\cdot}72\frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial\theta^2} \tag{16b}$$

$$D = -0{\cdot}28\frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial\theta^2} \tag{16c}$$

$$B = E = F = 0. \tag{16d}$$

In order to compare the present results with those of Quenby and Lietti (1968) we note that their results can be rewritten in the tensor form used here as

$$C-A = \frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial\theta^2} \tag{17a}$$

$$D-A = 0 \tag{17b}$$

$$B = E = F = 0. \tag{17c}$$

Comparing the results (16*a*–*d*) and (17*a*–*c*), it can be seen that (with minor differences) the results of the two models are basically in agreement.

Nagashima *et al* (1971) have suggested that the observed second harmonics of the free space anisotropy could be interpreted as a pitch angle distribution around the interplanetary field. In tensor form this means

$$C = D \tag{18a}$$

$$B = E = F = 0. \tag{18b}$$

Inspection of equations (15*a*–*f*) shows that this could be the case if equation (14) did not hold, but to the contrary, a sunward streaming existed along the magnetic field lines. The existence of such a sunward streaming has been suggested recently by Dyer *et al* (1973). (Although comparison of (15*c*) and (15*d*) shows that C cannot be exactly equal to D, $C = D$ can hold approximately if the term containing $S_{\parallel}$ is dominant.)

4. Diurnal and semi-diurnal variations

Due to the rotation of the earth the anisotropic free space cosmic ray angular distribution manifests itself in intensity variations in earth-based measurements. Variations caused by the second harmonics of free space anisotropy will be investigated in this section. Here we consider free space variations only and atmospheric and geomagnetic effects will not be taken into account. Variations resulting from the first harmonics of the anisotropy are given elsewhere (cf Somogyi 1972) and will be disregarded here.

When calculating numerical results the figures given by Quenby and Lietti (1968) will be adopted, ie

$$\frac{R^2}{2r^2}\frac{1}{U}\frac{\partial^2 U}{\partial \theta^2} = 0{\cdot}001P\%$$

at the orbit of the earth where P is the rigidity (in GV), $\psi = 45°$, and $\lambda_1 \propto r^2$ (ie $\beta = 2$ and $\eta = 0{\cdot}22$) will be assumed (see equation (14)). As high rigidities are of interest $\lambda_2/\lambda_1 = \frac{1}{3}$ will be taken (see equation (5)).

The following notations will be used:

- χ the declination of the axis of the earth ($\chi = 23{\cdot}5°$)
- Λ geographical latitude of the asymptotic arrival direction of cosmic ray particles observed
- t solar time: $t = 0$ at midnight and $t = 180°$ at noon
- α time defined by the position of the earth in its orbit: $\alpha = 0$ on 21 December and $\alpha = 360°$ in a year

4.1. *Annual and semi-annual variations*

Because of the 23·5° declination of the axis of the earth, the second harmonics of the cosmic ray anisotropy cause a semi-annual variation. Calculation gives

$$\Delta J/J = -\sin\chi\cos\chi(1-3\sin^2\Lambda)[E\cos(\alpha-\psi)-F\sin(\alpha-\psi)] \\ -\sin^2\chi[(1-3\sin^2\Lambda)/2][\tfrac{1}{2}(A-C)\cos 2(\alpha-\psi)-B\sin 2(\alpha-\psi)]. \tag{19}$$

Using equations (16*a*–*d*) at the equator at 20 GV rigidity the annual variation turns out to be non-existent ($E = F = 0$), while a semi-annual variation of amplitude about $0{\cdot}18\times10^{-4}$ is to be expected with intensity maxima on 5 May and 5 November.

4.2. Diurnal variation

Transforming the second harmonics of the free space anisotropy into geographical coordinates, a diurnal variation is obtained:

$$\begin{aligned}\Delta J/J = \sin\Lambda\cos\Lambda\{&(A+C-2D)\sin\chi\cos\chi\cos(t+\alpha)\\ &+\sin\chi[(1+\cos\chi)/2][(A-C)\cos(t-\alpha+2\psi)+2B\sin(t-\alpha+2\psi)]\\ &-\sin\chi[(1-\cos\chi)/2][(A-C)\cos(t+3\alpha-2\psi)-2B\sin(t+3\alpha-2\psi)]\\ &+(2\cos\chi-1)(1+\cos\chi)[E\cos(t+\psi)+F\sin(t+\psi)]\\ &-(2\cos\chi+1)(1-\cos\chi)[E\cos(t+2\alpha-\psi)-F\sin(t+2\alpha-\psi)]\}. \qquad (20)\end{aligned}$$

Combining this with results (16*a*–*d*), the amplitude of the diurnal variation turns out to have annual modulation with maximal amplitude being $0{\cdot}8\times10^{-3}$ at $\Lambda = 45°$ and $P = 20$ GV. Figure 2 shows the diurnal harmonics dial for the present results and for those of Quenby and Lietti (1968). The results obtained from the two models are slightly different.

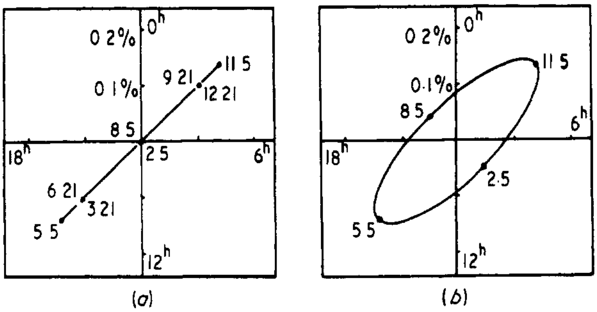

Figure 2. Annual change of the expected harmonic dial of diurnal variation caused by the second harmonics of cosmic ray anisotropy for (*a*) the model of Quenby and Lietti and (*b*) present calculations. $\Lambda = 45°$ and $P = 20$ GV are chosen. Numbers indicate months and days.

4.3. Semi-diurnal variation

The second harmonics of the free space anisotropy result in a semi-diurnal variation of

$$\begin{aligned}\Delta J/J = \tfrac{1}{2}\cos^2\Lambda\{&[(1+\cos\chi)/2]^2[(A-C)\cos 2(t+\psi)+2B\sin 2(t+\psi)]\\ &-\tfrac{1}{4}\sin^2\chi(A+C-2D)\cos 2(t+\alpha)\\ &-[(1-\cos\chi)/2]^2[(A-C)\cos 2(t+2\alpha-\psi)-2B\sin 2(t+2\alpha-\psi)]\\ &-\sin\chi(1+\cos\chi)[E\cos(2t+\alpha+\psi)+F\sin(2t+\alpha+\psi)]\\ &+\sin\chi(1-\cos\chi)[E\cos(2t+3\alpha-\psi)-F\sin(2t+3\alpha-\psi)]\}. \qquad (21)\end{aligned}$$

Inspection of this result shows that each of the five independent components of the quadrupole moment of the cosmic ray directional distribution manifests itself in semi-diurnal variation. Because of their different seasonal variations, in principle at least, it is possible to determine them separately.

Using the results of equations (16*a*–*d*), the semi-diurnal variation turns out to be nearly constant with times of intensity maxima at about 3 hr and 15 hr local solar time. This result is in agreement both with the prediction of the model of Quenby and Lietti (1968) and with most of the experimental results (cf Rao and Agrawal 1970, Kargathra and Sarabhai 1971, Dutt *et al* 1973). The amplitude of the semi-diurnal variation in the present model has a small ($\simeq 10\%$) semi-annual variation. The results obtained from the present model and from those of Quenby and Lietti are shown in figure 3. The discrepancy between the two models again turns out to be rather small.

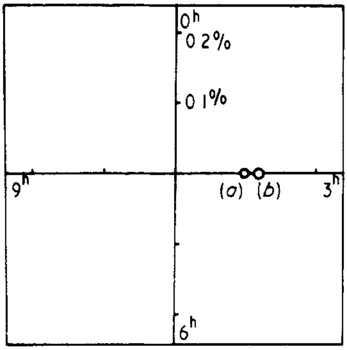

Figure 3. Expected harmonic dial of semi-diurnal variation ($\Lambda = 0°$, $P = 20$ GV) for (*a*) the model of Quenby and Lietti and (*b*) present calculations. The semi-diurnal amplitude has a slight semi-annual variation with maximal values on 5 February and 6 August.

5. Conclusions

In order to describe the semi-diurnal variation the convection–diffusion theory has been extended by considering the second moments of the cosmic ray angular distribution and those of the statistical Boltzmann equation. The quadrupole moment of the directional distribution has been represented by a symmetric traceless tensor (equation (2)) whose elements correspond to the five independent spherical harmonics of second order. The main features of the calculation can be summarized as follows.

(i) Different λ_1 and λ_2 mean free paths belong to the first and second harmonics of the anisotropy, ie the rates at which different harmonics decay may be different. The quadrupole moment of the cosmic ray angular distribution turns out to depend on the λ_2/λ_1 ratio (see equations (15*a*–*f*)). At high rigidities $\lambda_2 = \frac{1}{3}\lambda_1$ is to be expected (equation (5)). At low rigidities, however, the quadrupole moment of the anisotropy may provide information on the ratio of the two mean free paths, ie on the nature of the magnetic field irregularities by which the particles are scattered.

(ii) The present results are basically in agreement with those of Quenby and Lietti (1968), although quantitative predictions may differ by 10–20%. There are two marked differences as well:

(*a*) By contrast with Quenby's model, different particle fluxes are predicted from the directions along the magnetic field and normal to the ecliptic plane respectively. This basically implies the fact that the mean square distance from the ecliptic plane is larger for particles arriving from the latter direction.

(*b*) Since the interplanetary magnetic field lines are bent and diverging, the second harmonics of the anisotropy depend on the global feature of the mean free path λ_1 between the sun and the earth (see factor η in equations (14), (15*a*–*f*)).

(iii) Provided that cosmic ray density depends on heliolatitude it can readily be seen that the cosmic ray angular distribution does depend on directions perpendicular to the magnetic field (ie $D \neq C$, see equations (15*c*, *d*)). $C = D$ should hold if the cosmic angular distribution were dependent on the pitch angle only. The pitch angle distribution is a good approximation if the terms produced by the streaming along the magnetic field are dominant, ie a considerable sunward streaming exists. In fact there is some indication in favour of such an inward streaming along the magnetic field (Dyer *et al* 1973).

Acknowledgments

The author is greatly indebted to Professor A J Somogyi and Dr T Gombosi for stimulating discussions. Drs J J Quenby and S Webb are thanked for many illuminating remarks.

References

Dutt J C *et al* 1973 *Proc. 13th Int. Conf on Cosmic Rays, Denver*, vol 2 (Denver: University of Denver) pp 1011–5

Dyer C S *et al* 1973 *Proc. 13th Int. Conf. on Cosmic Rays, Denver*, vol 2 (Denver: University of Denver) pp 1329–34

Gleeson L J and Axford W I 1967 *Astrophys. J.* **149** 115–8

Jokipii J R 1966 *Astrophys. J.* **146** 480–7

Kargathra L V and Sarabhai V A 1971 *Proc. 12th Int. Conf. on Cosmic Rays, Hobart*, vol 5 (Hobart: University of Tasmania) pp 1879–84

Nagashima K *et al* 1971 *Proc. 12th Int. Conf. on Cosmic Rays, Hobart*, vol 2 (Hobart: University of Tasmania) pp 661–5

Parker E N 1964 *Planet. Space Sci.* **12** 735–49

Quenby J J 1973 *Proc. 13th Int. Conf on Cosmic Rays, Denver, Invited and Rapporteur Talks* (Denver: University of Denver) pp 3731–50

Quenby J J and Hashim A 1969 *Planet. Space Sci.* **17** 1121–39

Quenby J J and Lietti B 1968 *Planet. Space Sci.* **16** 1209–19

Rao U R and Agrawal S P 1970 *J. Geophys. Res.* **75** 2391–401

Somogyi A J 1972 *Acta Phys. Hung.* **32** 261–74

Subramanian G and Sarabhai V 1967 *Astrophys. J.* **149** 417–28